

## **Context in application and modelling - an empirical approach**

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### *Abstract:*

In a qualitative-orientated empirical study the effect of context on the students' approach towards real world problems is investigated. First results show that contexts given in a task can internally be reconstructed in very different ways. It is also shown that both, the student's affective proximity to the context and the student's substance knowledge can have fostering but also hindering effects on his or her performance.

## **1 Introduction**

The theme of the paper is context, a rather blurring concept, used by many authors in different meanings and ways. Sometimes even different concepts of context can be found in the same study. Also several names are given for this concept such as cover story, situation, situational context, setting, social context etc.

On the other hand it is consensus that context - however it is meant - is a central topic in the discussion about modelling and application in mathematics education. Since its very beginning the debate on the teaching of mathematical modelling and applications claims the usage of various contexts or contextualized problems. We associate many goals with the usage of contextualized problems, among others the fostering of motivation for the learning of mathematics or as a way to visualize mathematical concepts and methods (Kaiser, 1995). Until now we do not have comprehensive empirical evidence how far our hopes concerning the usage of contextualized problems are fulfilled.

The empirical study we are going to present tries to give a few answers to this problem, although we are far away from having final answers. First we will clarify the meaning of context. Afterwards some results from previous empirical research studies are presented. In the third part the aim of our own study and the methodical design will be described. In the fourth part first observations are detailed, finally in the fifth part first results of the influence of the so-called *subjective figurative context* will be described.

## 2 Research Results from Previous Studies

Clarke & Helme (1996) propose to distinguish between *figurative context* and *interactive context*. *Figurative context* comprises the real scenario the task is embedded in. *Interactive context* describes the conditions the task is encountered in by a student. Applying a constructive perspective they point out that both kinds of context are individually constructed.

Following this idea in our study we distinguish between

<i>objective figurative context:</i>	description of the real scenario given in the task
<i>subjective figurative context:</i>	individual interpretation of the <i>objective figurative context</i>

Although a similar distinction is theoretically also possible for the *interactive context*, we avoid this distinction for reasons of clarity since the focus of this paper is figurative context.

It is presumably the *objective figurative context* that is often implicitly meant by researchers when referring to "context".

Objective figurative contexts are generally chosen so that they are presumably near to students. Generally this is linked to the expectation of an fostering effect for the problem-solving process. However, which effect the objective figurative context offered in the task does have on a student depends on the way he or she constructs his or her own *subjective figurative context* (see as an example the story about the dog Nadja in section 3). The effects of an objective figurative context has not been investigated comprehensively yet, the studies carried out so far do not provide an uniform picture.

In the research results of almost all of the empirical studies on context the term *figurative context* is not used by researchers, rather many different expressions are used - among others the word context. For the sake of clarity we use the term *figurative context* in the meaning of *objective figurative context* in this paper unless stated otherwise.

The culture of the mathematics classroom seems to be a factor. Boaler (1993) found in her comprehensive study that in an open, discussion-centred mathematics-classroom the figurative context had a smaller effect than in traditional classrooms where figurative contexts seem to influence the students' performance. But no general pattern could be found which figurative contexts have a fostering and which have a hindering effect.

According to Stillman (1998) the degree of involvement with the figurative context is influenced by the task type. So-called "wrappper problems" provoke less, modelling-tasks more involvement with the figurative context. Generally she found that medium to high involvement with the figurative context leads to better performance. But there are exceptions from this tendency, especially concerning students who perform generally well in mathematics.

When investigating the problems minority students in the Netherlands have with contextualized tasks Carvalho de Figueirido (1999) found out, that language and cultural background have an effect on the way figurative contexts are understood. It was also found that illustrations can mislead students in their understanding of the figurative context of a task. Furthermore it was pointed out that an unfamiliar figurative context tempt students to de-contextualise the task fast.

In some research studies not presented here *familiarity* with the figurative context is mentioned as a fostering effect on the students' performance. But this is not generally confirmed in other studies.

To sum it up: In all the studies mentioned it remains an open question how the figurative context offered in the tasks (by words or illustrations) is represented internally and which effects it shows in the course of the solution process.

These questions were the central questions of our own qualitative-orientated study.

### 3 Aim of the study and methodical design

The aim of the study is the generation of hypotheses about the effects the figurative context can have on secondary school students. We are aiming at a development of a categorization of the effects observed.

This study uses the case study approach, therefore generalizations cannot be done easily and have to reflect the special characteristics of the sample and the observation design.

The development of the methodical design of the study was rather difficult, because we needed methods which allowed us to reconstruct the internal representations of the context by an individual student and which gave us insight into the emotional reactions of the student. The problem was that no direct observation or measurement was possible.

We carried out some pre-studies. Among other approaches a group of students was asked to write free stories about the task immediately after they had worked on it. This was done with 12- and 19-year-old students.

Let us give an example of the story a twelve year girl wrote after working on the following task:

A dog's first year is equivalent to 15 human years. The second year of a dog is equivalent to 6 human years. Each of the following years of a dog is equivalent to 5 human years.

(After this introductory text some calculation-tasks were given.)

After solving the task correctly the girl wrote:

I have a dog named Nadja. She was born in 1986 and her age is 13 dogyears, which is approximately 76 human years. Anyway, when I was approximately 2 years old I was sitting on the baby's changing unit. I could not feel my legs from the first day on. But somehow, nobody knows how, I tried to crawl. Nadja heard that and came running into the living room. I fell down. Fortunately Nadja was there and I fell on Nadja's back and then onto the floor. Up to the age of 3 I could not feel my legs! All that is true!!!

Independent from the truth of the story this is an impressive and touching example what can be on a student's mind when confronted to a task. This observation, among others,

confirmed our conjecture that figurative contexts can have very individual, unpredictable effects. This reinforces the necessity of the concept of a *subjective figurative context* we have mentioned in the beginning.

As a result of the pre-studies the following three step design appeared to be the best: Four pairs of students (16 to 17 years old, in their 1<sup>st</sup> year of upper secondary school, two pairs of boys, two pairs of girls) from four different schools were given realistic tasks. The students were videotaped while working on the tasks. After the work on the task the students watched individually the video-record. They were requested to stop the tape when during the solution-process something concerning the figurative context crossed their mind. They were asked to utter these thoughts. The statements were audiotaped. The interviewer could as well stop the record if the student did not do so while the interviewer considered certain parts of the record as potentially useful for statements. This method is known as method of stimulated recall (e.g. Weidle and Wagner, 1994). In an immediately following interview the student was asked deepening questions about the statements made during the stimulated-recall-phase.

The purpose of the design was to catch thoughts, associations etc. concerning the figurative context in that moment in which they crossed the student's mind ("in statu nascendi") without disturbing the solution process. The statements made during the stimulated-recall-phase were usually not interrupted by the interviewer to avoid disturbances. The place and time for further-going questions was the interview.

The data sampling was finished early 2001, not all of the 24 data-sets have been transcribed yet. These days we are in the process of a first systematic approach towards the data. So what we can present are first results far from being final. Especially no generalizations of or remarks on the quantitative spread of the phenomena observed can be made.

The tasks were given on three different days. The tasks were matched to different objective figurative contexts. The tasks differed also in the degree mathematical modelling is needed. The mathematical theory required for all the tasks was taken from former years of the students' maths-education. There are two reasons for that:

- It was not clear which topics had already been covered at the different schools.
- The mathematics should remain in the background for the benefit of the

figurative context. In other words: The maths was supposed to be well known.

The first task which was given to the students is shown below. The text is translated from German. Before we refer to the students' results let us comment the task briefly:

- The data are authentic, they were taken from publications of an oil-company. The students had not been informed about the authenticity of the data before they were working on the tasks.
- One might be tempted to solve the task by applying a mere extrapolation. But in order to determine the consumption in a certain number of years it is necessary to cumulate the consumption figures. This is the main mathematical difficulty of this task.
- The global consumption seems to be almost linear. Indeed if the results of a linear approach are compared with those of an exponential one only little difference is found. Using a linear model one gets a range of 32 years, the exponential approach leads to a range of 31 years. (It might be interesting to know that these figures meet well with predictions made by geo-scientists.)

>INSERT TABLE 1 ABOUT HERE<

#### **4 Reactions of the students on the task**

The task was almost acceptably solved by one pair by using a table. Two pairs used a promising approach but failed in applying the mathematical techniques properly. One pair did not really understand the task.

We do not want to present the answers rather than giving an outlined reconstruction of the statements made about the figurative context. We restricted the number of students in this paper for the sake of clarity. Future publication will show more detailed results.

In the transcripts we present many filling words were omitted for reasons of better understanding, which is shown by dots in brackets. Underlined words were spoken with emphasis. A question mark indicates, that the spoken word could not be heard properly.

**Christine** almost completely ignored the figurative context of the task. Only in the

beginning she mentioned a task that appears to be similar for her. This task was about cell growth. She tried by adapting known and looked-up formulae to the task to come to a result. She accepted her result of a range of approximately 361 years without any comment. (Remember: a correct answer would be approximately 32 years.)

During the stimulated recall phase she made no statements about the figurative context (in contradiction to the instructions given) but explained her way of working on the task focussing on technique.

In the interview Christine was asked questions about the figurative context:

*Interviewer:* Have you ever dealt with this topic in another school subject?

Or in another way through newspapers or-

*Christine:* Hm- for which topic?

*Interviewer:* What this is about here.

*Christine:* Mineral oil? Oh, you mean mineral oil?

*Interviewer:* If you think mineral oil is the topic, so that's what I mean.

*Christine:* Ehem. No. [laughing]

Later in the interview she said:

*Christine:* Mineral oil always sounds so uninterestingly.

So for Christine the figurative context hardly seemed to be existent when working on the task. Even later in the interview she seemed to think for the first time that the task even has a figurative context. Then (and not during the solution process) she gave contextual comments.

**Ingo** associated more than Christine. His first thought was the driving licence test his sister had passed on the same day. He reflected his personal situation in the future when oil would be used up. He had an idea about the value of the range until oil will be used up and said that his knowledge was based on media-publications. He talked about alternative technologies which could extend the range of the oil-reserves and also mentioned a long discussion with his chemistry-teacher about the subject. The oil crises and the gulf war was mentioned by him and he said in the interview that he was concerned about the increasing oil-consumption:

*Ingo:* At any rate I also would worry about, and I think as it is mentioned in

the media, it is interesting for every student of my age how it will be in future. (...) And (...) well, I personally worry about that; well, I have already thought about it quite often. That's really in my mind. The point where it will be finished.

He had some problems in applying mathematical techniques properly, so he failed very close to an successful completion of the task.

**Karla** associated in a emotional way the problem of destruction of natural environments. Her chemistry lessons provided her with knowledge about how oil comes into being, the limitations of natural resources and other ecological problems. She liked chemistry lessons and her chemistry teacher, which she described in the interview as follows:

*Karla:* Yes that was a topic I am interested in for its ecological aspects. And also chemistry is a school subject I like; I make out well with my chemistry teacher. (...) and therefore, it has been an important topic to me, also in that lesson. And I liked it, and we watched such a film about when energy resources will be exhausted and (...) that actually did interest me and concerned me I would say.

Karla reflected the assumption of a continuously increasing oil consumption and was rather considering a decreasing oil consumption. On the background of the emotional importance the problem of the destruction of natural environment had for her, one is almost tempted to interpret this as a wish. She said in the stimulated recall:

*Karla:* (...) yes, there one notices that the earth is already quite wrecked, and (low voice) in this respect it actually interested me and (louder) then I thought about it again in that situation. (...) if really every year zero point zero seven (...) billion tons more are consumed or if it is perhaps only for a while or (...) if one really may say it that way, that it always increases or may be it was (only?) (...) for three or five years like that (...) and that then it will decrease again or (...) what might happen somehow to solve the ecological problem (...).

Karla solved the task by using tables satisfyingly with only one little mistake.

Let us summarize: It becomes obvious that different students interpreted the objective figurative context offered in the task in very different ways. The differences appeared in the associated aspects of reality and in emotional attitudes.

It must be mentioned that the students' statements do of course not cover completely their internal representations of the figurative context offered in the task. One has to expect that many thoughts and associations remain unmentioned due to either missing consciousness of the thoughts or to a missing will to tell them.

In spite of the just mentioned limitations the differences in the perception and interpretation of the objective figurative context offered in the tasks are obvious. These differences are based on individual knowledge, experience and attitude. Consequently reflections on the effects of figurative contexts must consider its individual nature.

## **5 Influence of the subjective figurative context**

In the following we want to present first results of possible contextual influences gained by the reconstruction of the *subjective figurative contexts*.

### *5.1 The subjective figurative context can cause a distraction from the task.*

Although generally contextualized tasks are seen as helpful, distracting effects could also be observed. For example Ingo mentioned in the interview:

*Ingo:* (...) somehow has reminded me of statements and such things I read about. (And?) then (...) at first I had to shake off this thought, so that I could concentrate (...) again (on the?) task because (...) for a moment I was tempted to think too much about it.

*Interviewer:* It would have disturbed you?

*Ingo:* (answering quickly) Yes.

*Interviewer:* Because it-

*Ingo:* Because (...) it does not help us to solve the problem and because then it is difficult to concentrate again on the problem. Because then one has another thought at the same time, and of course one associates personal experiences or what one has read. If one thinks too much about it, it is more

difficult, then one must come back (...) to the problem.

This phenomenon of distraction can have different aspects. So an emotional and a more matter-of-fact-related distraction could be found. Let us present an example for each of the two aspects mentioned.

*5.1.1 Emotional involvement in the figurative context can superpose or even disturb the work on the task.*

This is remarkable because usually it is expected that familiarity with or interest in the figurative context of the task has a helpful effect. The opposite seems to be possible as well.

The above mentioned student Karla who showed a very emotional involvement said in the interview that during the work on the task she always had to think about the films about environmental destruction shown in the chemistry lessons. Asked if she felt disturbed by that she answered:

*Karla:* Yes, may be, it happens often to me that somehow I totally have to think of different things while solving problems, also during tests. (...) And then I say to myself concentrate and so and then it works, but in that moment one thinks about these films and so and then one does not think about the task and on how to solve it. That is a different problem.

This is interesting because there exist didactical approaches to support especially girls by using figurative contexts which are assumed to be close to girls. Further-going analyses have to be done in this field.

*5.1.2 A rich subjective figurative context can be a disturbing factor when working on the task.*

In a certain sense a rich subjective figurative context produces a world too large for the more or less narrow task. The subjective figurative context contains more information than the text of the task itself and so the student has to choose what is important and what is not.

During the problem-solving process and in the stimulated-recall-phase the student Arthur mentioned a large variety of real-world-factors that could influence the global oil consumption. Asked about that in the interview:

*Interviewer:* Which influence did these thoughts have (...) on you while working on problems? Can you tell it?

*Arthur:* (...) (low voice) while solving the problem they only made it more difficult because it was (?) difficult to estimate. I (somehow?) assumed a difference of ten years. That is mathematically hardly solvable.

*Interviewer:* Well, I understand that you were thinking so many things how complex it is-

*Arthur:* Yes.

*Interviewer.* And that has disturbed you?

*Arthur:* (answering quickly) Yes.

*Interviewer:* Hm. Okay.

*Arthur:* Well, I mean it has made me thoughtful because if (...) one imagines how it might be, the problem cannot be solved. If one is given (a formula?) how long it might last. I think after being given the formula a task could be solved easier. But like this it was (low voice) too (...) big field to (?) simply.

Here a distraction from the task caused by too much content information and not by emotional involvement as described above takes place.

Again this seems to be paradox: A task which is meaningful to the student becomes more difficult due to the host of associations.

*5.2 Frequently the subjective figurative context is used to control whether a result is plausible. In conjunction with a misconception of the modelling process this can cause an error.*

In the case we have in mind the student Josef (Arthur's partner) predicted the global oil consumption for the year 2000 *lower* than the consumption for 1999. They used the mean of the consumption-figures for the years 1995 to 1999 as a prediction for 2000. After feeling that this might be incorrect the student finally accepted this result giving the following reason: The result can be correct because the lately introduced German

ecological tax leads to a decreasing oil consumption.

Later asked in the interview how he would argue if the figurative context was ore deposits the student said in this case he would not accept the result because he did not know anything about ore deposits.

How can this be understood? The following explanation is possible: Josef had - probably not consciously - build and evaluated a mathematical model. After the evaluation (and not, as it would be correct, while building the model) he added an additional figurative-context-assumption, the introduction of the ecological tax. So he justified a result, which appeared not to be correct at the first sight.

He did not, as it would be correct, start another circle of the modelling process including the new assumption of the introduction of ecological taxes. Here a kind of modelling-error took place, to which the student was tempted by the figurative context of the task. It becomes clear, how important it is to talk about mathematical modelling in the mathematics-classroom also on a meta-level to avoid mistakes like this.

### 5.3 *The subjective figurative context can cause motivation.*

Although this seems to be common-sense it should be mentioned here after all the negative effects we mentioned earlier in this paper. Motivation is an effect which is sometimes taken as the only one. It remains an open question at the moment how strong the motivational effect is, if it carries through the work on the task or if it is just a mind-opener in the sense that it serves as a catcher.

The above mentioned student Ingo stated in the interview:

*Ingo:* [...] had the (...) problem without theme, just purely mathematical, well, naked, then for instance I would have been sitting down and thinking no, don't feel like doing it. Well, in some way I would not have gone into the problem, very probably. (...) That this topic has helped me very much to find a way for getting into the mathematics. Otherwise I would never have come to this point where I worked with enthusiasm on it.

## 6 Final remarks

The hopes concerning the inclusion of contextual problems such as the fostering of motivation and understanding claimed in the theoretical debate on applications and modelling have to be seen in a more differentiated light. We have to consider that the figurative context might have a variety of influences, which might be fostering or hindering. We even have to consider that the same figurative context might have different effects on a student dependent on his or her foregoing experiences or the situation the student is in. We have to be more careful with psychological arguments concerning the inclusion of modelling and applications in mathematics education. We should put more emphasis on normative reasons for the inclusion of real world problems in mathematics education. Furthermore consequences for teacher education and in-service-training have to be taken: We need to make student-teachers and practising teachers more aware of the students' various individual reconstructions of the figurative context offered in a task and the possible influences a figurative context can have on the students.

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TABLE 1

By the end of the year 1999 the global oil reserves were estimated to approximately 138.041 billion tons.

Over the past years the global oil consumption has been increasing permanently.

The consumption figures from the past can be taken from the diagram opposite.

- a) Determine the amount of global oil consumption that approximately can be expected for the year 2000.
- b) When approximately will the reserves be used up?



